

(i) Printed Pages : 3

Roll No. ....

(ii) Questions : 9

Sub. Code : 

0	2	4	1
---	---	---	---

Exam. Code : 

0	0	0	3
---	---	---	---

B.A./B.Sc. (General) 3<sup>rd</sup> Semester  
1128

MATHEMATICS

Paper : I (Advanced Calculus-I)

Time Allowed : Three Hours]

[Maximum Marks : 30

Note :— Attempt any five questions in all selecting at least two questions each from Unit-I and Unit-II each question carry equal marks.

UNIT—I

1. (a) Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$f(x, y) = x \sin \frac{1}{y} + y \sin \frac{1}{x}, \quad x \neq 0, y \neq 0. \text{ Prove that}$$

$$\lim_{(x, y) \rightarrow (0, 0)} f(x, y) = 0.$$

(b) Discuss the continuity of the following function at  $(0, 0)$  :

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

2. (a) If  $z(x + y) = x^2 + y^2$ , then show that

$$\left( \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2 = 4 \left( 1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right).$$

0241/EPY-10109

1

[Turn over

(b) Show that  $f_{xy}(0, 0) \neq f_{yx}(0, 0)$ , where

$$f(x, y) = \begin{cases} x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\frac{x}{y} & \text{if } xy \neq 0 \\ 0 & \text{if } xy = 0 \end{cases}$$

3. (a) If  $u = e^x \sin y$ ,  $x = \log t$ ,  $y = t^2$ , then by partial differentiation find  $\frac{du}{dt}$ . Also verify by direct calculations.

(b) Show that  $f(x, y) = \cos(x + y)$  is differentiable at  $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ .

4. (a) Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at the point  $(2, -1, 2)$ .

(b) Define irrotational vector. Find constants  $a, b$  and  $c$  for which  $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$  is irrotational.

### UNIT—II

5. (a) If  $z = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ , prove that :

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = -\frac{1}{2} \cot z.$$

(b) Use Taylor's theorem to expand  $x^2 y + 3y - 2$  in powers of  $x - 1$  and  $y + 2$ .

6. (a) If  $u^3 + v + w = x + y^2 + z^2$ ,  $u + v^3 + w = x^2 + y + z^2$ ,  $u + v + w^3 = x^2 + y^2 + z$ , prove that :

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{1 - 4(yz + zx + xy) + 16xyz}{2 - 3(u^2 + v^2 + w^2) + 27u^2 v^2 w^2}.$$

(b) Show that the functions  $u = \frac{x}{y-z}$ ,  $v = \frac{y}{z-x}$  and

$w = \frac{z}{x-y}$  are not independent of each other and also find

the relation between them.

7. (a) Find the envelope of a system of concentric and co-axial ellipses of constant area.

(b) Show that the evolute of the curve

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}} \text{ is } (x+y)^{\frac{2}{3}} + (x-y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}.$$

8. (a) Find all the points of maximum and minimum of the function:

$$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy.$$

Also discuss the saddle points of the function.

(b) Find the maximum and minimum value of  $x^2 + y^2$  subject to the condition  $3x^2 + 4xy + 6y^2 = 140$ .

## Facing Problems in Graduation Math We are here to guide you.

We are here to help you. Mathematics as a subject in graduation is a challenging task for many students. We focus on basics and art of problem solving rather than just learning the solution part.

### Doubt Solving:

Our Doubt Clearance Sessions, emphasizing Misconceptions and repeated errors. Along with that, we help you create study plan that will ensure your success in exam.

### Foundation for IIT JAM & CSIR NET and Competitive Maths

Mathematics in Graduation is first step towards the higher and competitive Maths. We focus on conceptual understanding which work as a base for the competitive maths.

### Clear B.Sc., B.Com, BCA, B.Tech College Maths with good grades

Getting good marks along with conceptual understanding will boost your confidence. Feel free to ask for Demo sessions. You have liberty to join the classes for specific topic rather than whole book.

### Dr. Himanshu Singla (Ph.D Maths)

7 Years Teaching Experience at prestigious institutes like NMIMS (Chd), DAVC Sector 10, PGGC 11 Chd., UIET Sector 25



## We make Maths easy for you at Abhyaskul. We are here to guide you.

We are here to help you. Mathematics as a subject in graduation is a challenging task for many students. We focus on basics and art of problem solving rather than just learning the solution part.

### Follow our You Tube Channel. (Search Abhyaskul at You Tube)

Learn the basics of math at our YouTube channel!

Our channel covers a wide range of math topics, from basic arithmetic to algebra and geometry. You can always ask to make a video on any topic. We wil try to provide it as soon as possible.

Whether you're struggling with a particular concept or just want to brush up on your math skills, our YouTube channel is a great resource. Visit our channel today and start learning!

### Career Opportunities in Maths

After graduating in maths, a world of opportunities opens up. You can pursue careers in finance, data science, engineering, research, or academia. Maths graduates are in demand for their analytical and problem-solving skills, and they can play a key role in driving innovation and solving complex challenges in a variety of industries. For any query regarding career opportunity in Math, feel free to meet.

### Dr. Himanshu Singla (Ph.D Maths)

7 Years Teaching Experience at prestigious institutes like NMIMS (Chd), DAVC Sector 10, PGGC 11 Chd., UIET Sector 25

